

Angles In Circles

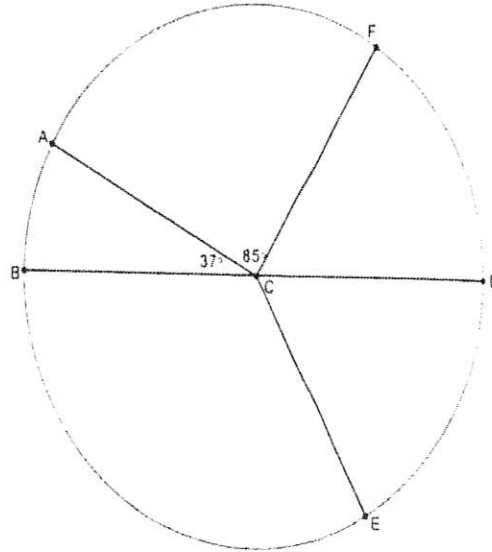
Part I – Central Angles

The measure of a central angle is _____ the measure of the intercepted arc created by the angle.

The total number of degrees in a circle is _____.

The total number of degrees in a semicircle is _____.

Use the picture of $\odot C$ at right for the example questions. Tell whether each arc is a major arc, minor arc, or a semicircle. Then, find the degree measure of each arc. BD is a diameter and CD is an angle bisector.



Ex 1: $m\widehat{AB}$

Ex 5: $m\widehat{BD}$

Ex 2: $m\widehat{AF}$

Ex 6: $m\widehat{FD}$

Ex 3: $m\widehat{BE}$

Ex 7: $m\widehat{DAE}$

Ex 4: $m\widehat{FBE}$

- 1.
- 2.
- 3.
- 4.
- 5.
- 6.
- 7.

Arc Length and Sector Area NOTES

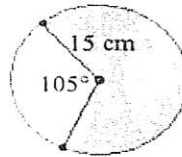
The area of sectors is actually finding a _____ of the circle's _____. You can set up a proportion to solve using $\frac{\text{part}}{\text{total}}$.

Ex 1.



_____ = _____

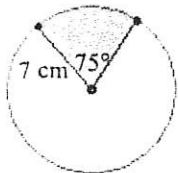
Ex. 2.



_____ = _____

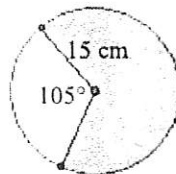
The arc length is actually finding a _____ of the circle's _____. You can set up a proportion to solve using $\frac{\text{part}}{\text{total}}$.

Ex 3



_____ = _____

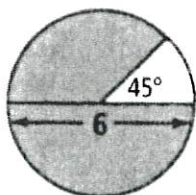
Ex. 4.



_____ = _____

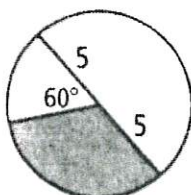
What about more difficult problems?

15.



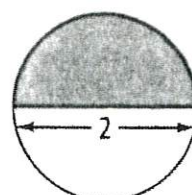
$\frac{63}{8}\pi$

16.



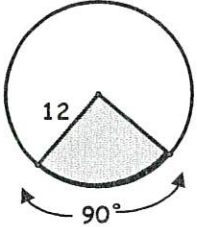
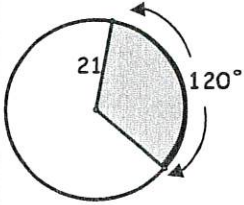
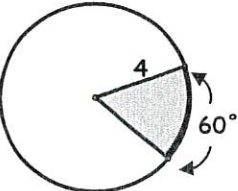
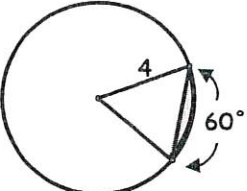
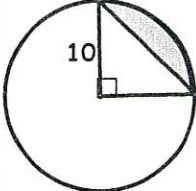
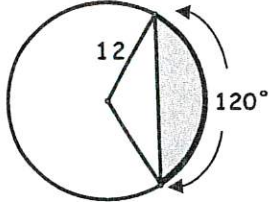
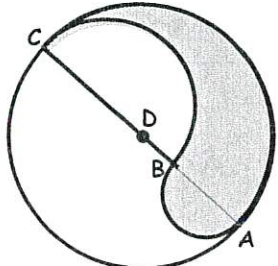
$\frac{25}{3}\pi$

17.



$\frac{\pi}{2}$

Find the shaded area. On problems 1-3, find the arc length for the shaded sector also.

<p>1. $A_{\text{sector}} =$ _____ Arc length = _____</p> 	<p>2. $A_{\text{sector}} =$ _____ Arc length = _____</p> 	<p>3. $A_{\text{sector}} =$ _____ Arc length = _____</p> 
<p>4. $A_{\text{segment}} =$ _____</p> 	<p>5. $A_{\text{segment}} =$ _____</p> 	<p>6. $A_{\text{segment}} =$ _____</p> 
<p>7. If $BC = 2AB$, what fraction of the circle is shaded? (Hint: Let the $AB = 2x$. D is the center of the big circle. AB is the diameter of a little circle and BC is the diameter of a medium circle. Find the areas in terms of x.)</p> 		
<p>8. Find the degree measure of the arc of a sector with area 36π if the area of the circle is 144π.</p>		
<p>9. Two circles have radii 3 cm. and 5 cm. Find the ratio of their areas.</p>	<p>10. The areas of two circles are in the ratio 16 to 9. Find the ratio of their radii.</p>	

Key

Angles In Circles

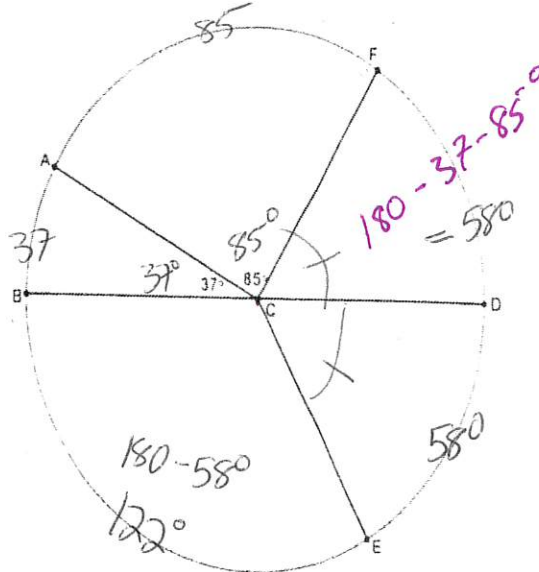
Part I – Central Angles

The measure of a central angle is equal to the measure of the intercepted arc created by the angle.

The total number of degrees in a circle is 360°.

The total number of degrees in a semicircle is 180°.

Use the picture of ⊙C at right for the example questions. Tell whether each arc is a major arc, minor arc, or a semicircle. Then, find the degree measure of each arc. BD is a diameter and CD is an angle bisector.




- minor Ex 1: $m\widehat{AB} = 37^\circ$
 - minor Ex 2: $m\widehat{AF} = 85^\circ$
 - major Ex 3: $m\widehat{BE} = 122^\circ$
 - major Ex 4: $m\widehat{FBE} = 244^\circ$
 - Semicircle Ex 5: $m\widehat{BD} = 180^\circ$
 - minor Ex 6: $m\widehat{FD} = 58^\circ$
 - major Ex 7: $m\widehat{DAE} = 302^\circ$
- $122 + 37 + 85 = 244$
 $360 - 58 = 302$

- 1.
- 2.
- 3.
- 4.
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- 6.
- 7.

Arc Length and Sector Area NOTES

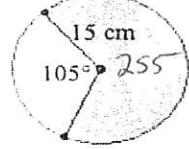
The area of sectors is actually finding a fraction of the circle's total area. You can set up a proportion to solve using $\frac{\text{part}}{\text{total}}$.

Ex 1. 

$$\frac{75}{360} = \frac{A}{\pi(7^2)}$$

$$49 \cdot 75 \cdot \pi = 360A$$

$$A = \frac{49 \cdot 75 \cdot \pi}{360} = 10.21 \pi \text{ cm}^2 \approx 32.07 \text{ cm}^2$$

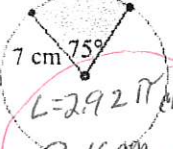
Ex 2. 

$$\frac{105}{360} = \frac{A}{\pi(15)^2}$$

$$A = \frac{105}{360} \cdot \pi(15)^2 = 500.69 \text{ cm}^2$$

$$A = 159.375 \pi \text{ cm}^2$$

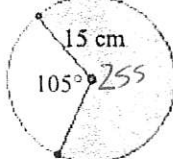
The arc length is actually finding a fraction of the circle's total circumference. You can set up a proportion to solve using $\frac{\text{part}}{\text{total}}$.

Ex 3. 

$$\frac{75}{360} = \frac{L}{2\pi(7)}$$

$$L = \frac{75}{360} \cdot 2\pi(7) = 9.16 \text{ cm}$$

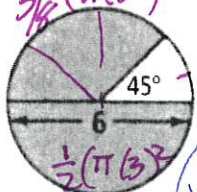
$$L = 14\pi = 43.98 \text{ cm}$$

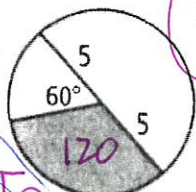
Ex 4. 

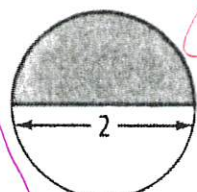
$$\frac{105}{360} = \frac{L}{2\pi(15)}$$

$$L = \frac{105}{360} \cdot 2\pi(15) = 21.25 \pi \text{ cm} \approx 66.76 \text{ cm}$$

What about more difficult problems?

15.  $\frac{45}{360} = \frac{A}{\pi(6)^2}$ $A = \frac{45}{360} \cdot \pi(36) = 4.5\pi$

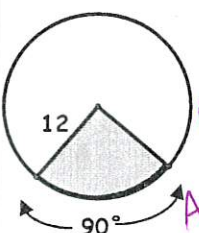
16.  $\frac{60}{360} = \frac{A}{\pi(5)^2}$ $A = \frac{60}{360} \cdot \pi(25) = \frac{5}{6}\pi$

17.  $\frac{90}{360} = \frac{A}{\pi(1)^2}$ $A = \frac{90}{360} \cdot \pi(1) = \frac{1}{4}\pi$

$\frac{63}{8}\pi = 27\pi + \frac{3}{8}\pi$
 $\frac{120}{360} = \frac{A}{\pi(5)^2}$ $A = \frac{120}{360} \cdot \pi(25) = 8.33\pi$
 $\frac{1}{2}(\pi(1^2)) = \frac{1}{2}\pi$

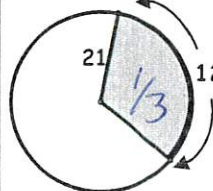
Find the shaded area. On problems 1-3, find the arc length for the shaded sector also.

1. $A_{\text{sector}} = 36\pi \text{ cm}^2$
Arc length = _____



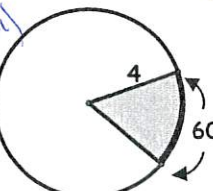
$A = \frac{1}{4}(2\pi(12))$
 $A = \frac{1}{4}\pi(12)^2$

2. $A_{\text{sector}} = 147\pi \text{ cm}^2$
Arc length = 14π



$A = \frac{1}{3}(\pi(21)^2)$
 $L = \frac{1}{3}(2\pi(21))$
 $\frac{120}{360} = \frac{12}{36} = \frac{1}{3}$

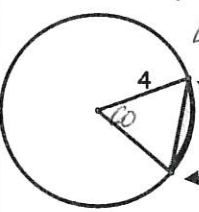
3. $A_{\text{sector}} = \frac{16}{6}\pi = \frac{8\pi}{3} \text{ cm}^2$
Arc length = $\frac{8\pi}{6} = \frac{4\pi}{3} \text{ cm}$



$L = \frac{1}{6}(2\pi(4))$
 $\frac{60}{360} = \frac{1}{6}$
 $A = \frac{1}{6}(\pi(4)^2)$

4. $A_{\text{segment}} = 2.67\pi - 6.93 = 1.646 \text{ cm}^2$

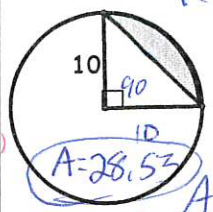
Sector: $A = \frac{60}{360} \cdot \pi(4^2) = 2.67\pi$



$\Delta = \frac{1}{2} \cdot 4 \cdot 4 \cdot \sin 60^\circ = 6.93$
 $A = 1.646 \text{ cm}^2$

5. $A_{\text{segment}} = \text{Sector} - \text{triangle}$

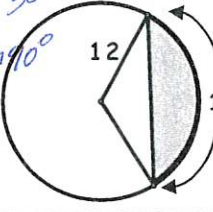
Sector: $\frac{1}{4}(\pi(10^2)) = 25\pi$



$\Delta = \frac{1}{2}(10)(10) \sin 90^\circ = 50$
 $A = 25\pi - 50$

6. $A_{\text{segment}} = 86.45 \text{ cm}^2$

Sector: $\frac{1}{3}(\pi(12^2)) = 144\pi/3$



$\Delta = \frac{1}{2}(12)(12) \sin 120^\circ = 62.35$

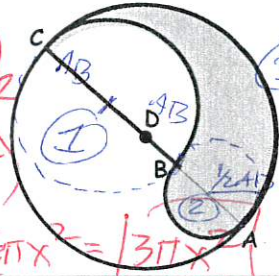
7. If $BC = 2AB$, what fraction of the circle is shaded? (Hint: Let the $AB = 2x$. D is the center of the big circle. AB is the diameter of a little circle and BC is the diameter of a medium circle. Find the areas in terms of x .)

Answer: $3\pi x$

$1/4 AB = 2x$
 $r_1 = 2x = AB$
 $r_2 = x = \frac{AB}{2}$
 $r_3 = 3(2x) = 3x = \frac{3(AB)}{2}$

Shaded = $\frac{1}{2}(\odot_3) - \frac{1}{2}(\odot_1) + \frac{1}{2}(\odot_2)$

$= \frac{1}{2}(\pi(3x)^2) - \frac{1}{2}(\pi x^2) + \frac{1}{2}(\pi x^2)$
 $= \frac{9\pi x^2}{2} - \frac{4\pi x^2}{2} + \frac{\pi x^2}{2} = \frac{6\pi x^2}{2} = 3\pi x^2$



8. Find the degree measure of the arc of a sector with area 36π if the area of the circle is 144π .

$\frac{360}{360} \cdot \frac{\theta}{360} = \frac{36\pi}{144\pi} \rightarrow \theta = \frac{36 \times 360}{144} = 90^\circ$

$\frac{36}{144} = \frac{1}{4}$

9. Two circles have radii 3 cm. and 5 cm. Find the ratio of their areas.

$\frac{\text{small}}{\text{large}} = \frac{9\pi}{25\pi} = \frac{9}{25}$

10. The areas of two circles are in the ratio 16 to 9. Find the ratio of their radii.

$\frac{\text{large}}{\text{small}} : \frac{\pi r^2}{\pi r^2} = \frac{16}{9} \rightarrow \sqrt{\frac{r^2}{r^2}} = \sqrt{\frac{16}{9}} = \frac{4}{3}$

Answers:

1. Area = $36\pi u^2$ and arc length = $6\pi u$

2. Area = $147\pi u^2$ and arc length = $14\pi u$

3. Area = $8\pi/3 u^2$ and arc length = $4\pi/3 u$

4. $\left(\frac{8}{3}\pi - 4\sqrt{3}\right)u^2$

5. $(25\pi - 50)u^2$

6. $(48\pi - 36\sqrt{3})u^2$

7. $\frac{1}{3}$

8. 90°

9. $\frac{9}{25}$

10. $\frac{4}{3}$

